General Random Effect Latent Variable Modeling: Random Subjects, Items, Contexts, and Parameters

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Overview

- Random loadings and variance models
- Cross classified structural equation models
- Cross classified structural equation models with random loadings and variance
- Random items, Generalizability Theory
- Examples of new features in Mplus Version 7, to be released late August
  - New multilevel features
  - New random effect features
- Bayesian estimation where ML and WLS are not feasible.
- The topics are covered more extensively in our August 27-29 course in Utrecht, see statmodel.com

Random loadings and variance models

Two-level factor model with random loadings

- y_{pij} is the $p-$th observation for person i belonging to cluster/group j. Group as a random mode.

$$y_{pij} = \mu_{pj} + \lambda_{pj}\eta_{ij} + \varepsilon_{pij}$$

- Measurement invariance often fails with large samples where all differences can be significant.
- The loading λ_{pj} can vary across clusters as random effects
- ML can estimate random effects for observed covariate but random effects for latent factors leads to numerical integration.
- Bayesian estimation straight forward even with categorical data

Two-level factor model with random loadings: simulation study with 5 indicators. ML v.s. Bayes.

Table: Absolute bias and coverage for factor analysis model with random loadings - comparing Bayes v.s. ML-Montecarlo

parameter	Bayes	Monte 500	Monte 5000
θ_1	0.00(0.97)	0.65(0.00)	0.42(0.01)
μ_1	0.01(0.95)	0.01(0.78)	0.00(0.80)
λ_1	0.01(0.96)	0.08(0.50)	0.04(0.60)
θ_2	0.02(0.89)	0.23(0.31)	0.15(0.50)
ψ_2	0.02(0.91)	0.10(0.23)	0.10(0.21)

Bayes is unbiased and has good coverage. ML is biased and has poor coverage.

The effect of treating random loadings as fixed parameters in continuous variables.

Table: Absolute bias and coverage for factor analysis model with random loadings - comparing random intercepts and loadings and v.s. random intercepts and fixed loadings models

parameter	Bayes	ML with fixed loadings
θ_1	0.00(0.97)	0.20(0.23)
μ_1	0.01(0.95)	0.14(0.66)
λ_1	0.01(0.96)	0.00(0.80)
θ_2	0.02(0.89)	0.00(0.93)

Ignoring the random loadings leads to biased mean and variance parameters and poor coverage. The loading is unbiased but has poor coverage.

The effect of treating random loadings as fixed parameters in categorical variables.

Table: Absolute bias and coverage for factor analysis model with categorical data and random loadings - comparing random loadings and intercepts v.s. random intercepts and fixed loadings models

parameter	Bayes	WLSMV with fixed loadings
τ_1	0.05(0.96)	0.17(0.63)
λ_1	0.03(0.92)	0.13(0.39)
θ_2	0.05(0.91)	0.11(0.70)

Ignoring the random loadings leads to biased mean, loading and variance parameters and poor coverage.

Random loadings with small number of clusters/groups

- Many applications have small number of clusters/groups. How many variables and random effects can we use?
- Independent random effects model - works well even with 50 variables (100 random effects) and 10 clusters
- Weakly informative priors are needed to eliminate biases for cluster level variance parameters
- Correlated random effects model (1-factor model) - works only when "number of clusters > number of random effects". More than 10 clusters are needed with 5 variables or more.
- What happens if you ignore the correlation: standard error underestimation, decreased accuracy in cluster specific estimates
- BSEM: Muthén, B. and Asparouhov, T. (2012). Bayesian SEM: A more flexible representation of substantive theory. Forthcoming in Psychological Methods.
- Using BSEM with 1-factor model for the random effects and tiny priors $N(1,\sigma)$ for the loadings resolves the problem.

Random loadings extensions: Cluster specific variance parameters

- In a two-level model every residual variable ε_{pij} can have a 1-factor model

$$\varepsilon_{pij} = \lambda_{pj}\xi_{pij}$$

$$Var(\xi_{pij}) = 1, Var(\varepsilon_{pij}) = \lambda_{pj}^2$$

- $\sqrt{Var(\varepsilon_{pij})}$ is normally distributed random effect
- For data with large cluster sizes, variance differences across groups are usually significantly different.
- In multilevel models the mean is typically cluster specific, why not the variance?
- Ignoring differences in variability between groups leads to errors in SE estimates.
- Mplus implementation needs a small residual such as 0.1 for ε_{pij}.

Random loadings extensions: Random loadings with group specific factor means.

- Model

$$y_{pij} = \mu_{pj} + \lambda_{pj}\eta_{ij} + \varepsilon_{pij}$$

$$\eta_{ij} = \eta_{w,ij} + \eta_{b,j}$$

- Group specific factor mean $\eta_{b,j}$
- If the random loadings are fixed parameters estimating a model with $\eta_{b,j}$ is equivalent to estimating a factor model on the between level.
- Correlated μ_{pj} can not be estimated simultaneously with $\eta_{b,j}$: both model between level covariance.
- To estimate correlated μ_{pj} and $\eta_{b,j}$ one can assume an anchor item using $Var(\mu_{1j}) = 0$, i.e., assuming mean invariance for the anchor.

Random loadings extensions: Random loadings with group specific factor means and variance.

- Model

$$y_{pij} = \mu_{pj} + \lambda_{pj}\eta_{ij} + \varepsilon_{pij}$$

$$\eta_{ij} = \eta_{w,ij} + \eta_{b,j}$$

$$\eta_{w,ij} = \sigma_j \xi_{ij}$$

- For identification purposes $Var(\xi_{ij}) = 1$
- Group specific factor mean $\eta_{b,j}$
- Group specific factor variance σ_j^2
- λ_{pj}, $\eta_{b,j}$ and σ_j are between level random effects.
- For identification purposes the mean of $\sigma_j = 1$

Random loadings extensions: General model where all parameters are cluster specific.

- Model

$$y_{pij} = \mu_{pj} + \lambda_{pj}\eta_{ij} + \varepsilon_{pij}$$

- Cluster specific intercept μ_{pj}
- Cluster specific loading λ_{pj}
- Cluster specific factor mean
- Cluster specific factor variance $Var(\eta_{ij})$
- Cluster specific residual variance $Var(\varepsilon_{pij})$

Random loadings extensions: Interaction between latent variables

- Random loading model allows the product of two latent variables.
- Using single level data with 1 observation in each cluster - computational trick
- Full information model with interaction terms between latent variables: $\eta_1 * \eta_2$
- Unlimited number of interactions (ML limited by dimensions of integration)
- It can be used with categorical or continuous variables.
- Extends to two-level models.

Random loadings extensions: Random EFA

- In multiple group / multiple time points EFA measurement invariance may not be full.
- We need EFA where all the loadings are group specific random effects.
- Estimate unrotated cluster specific loading model to get the cluster specific posterior for the unrotated solution
- Rotate the unrotated cluster specific posterior: one cluster and one MCMC iteration at a time.
- Cluster specific posterior distribution for the rotated solution.
- Methodology is based on Bayes EFA methodology: Mplus Version 7.

Random loadings example: multiple group factor analysis with non-invariant measurement model

- Student evaluation of teacher effectiveness, described in Marsh and Hocevar (1991).
- 35 items, 24158 observations, 21 groups
- Grouping based on the qualifications of the teacher and the academic discipline
- One factor model
- Measurement invariance does not hold: large sample size.
- Model modifications: not feasible with 21 groups

Testing for non-zero variance of random loadings

- Verhagen & Fox (2012) Bayesian Tests of Measurement Invariance. Submitted.
- Test the null hypothesis $\sigma = 0$ using Bayesian methodology
- Substitute null hypothesis $\sigma < 0.001$.
- Estimate the model with σ prior IG(1,0.005) with mode 0.0025

$$BF = \frac{P(H_0)}{P(H_1)} = \frac{P(\sigma < 0.001|data)}{P(\sigma < 0.001)} = \frac{P(\sigma < 0.001|data)}{0.7\%}$$

- $BF > 3$ indicates loading has 0 variance, i.e., loading invariance

Real data example - results

Table: Factor loading estimates for SEEQ data (variation range)

fixed	random means	random loadings	random loadings BSEM	σ	BF for $\sigma < 0.001$
0.76	0.79	0.81(0.67,0.94)	0.79(0.63,0.94)	0.005	0
0.78	0.81	0.82(0.71,0.92)	0.80(0.69,0.91)	0.003	0
0.80	0.80	0.82(0.71,0.93)	0.80(0.69,0.91)	0.003	0
0.73	0.74	0.76(0.67,0.85)	0.74(0.65,0.83)	0.002	3.4
0.88	0.83	0.85(0.70,1.01)	0.83(0.70,0.96)	0.006	0
0.89	0.83	0.85(0.74,0.96)	0.83(0.74,0.93)	0.003	0
0.84	0.75	0.77(0.64,0.91)	0.75(0.63,0.87)	0.005	0
0.90	0.87	0.89(0.83,0.95)	0.87(0.81,0.93)	0.001	16.9

Random loadings models show large range for standardized loadings and significant variance components for most loadings.

Random loadings in IRT

- Fox, J.-P., and A. J. Verhagen (2011). Random item effects modeling for cross-national survey data. In E. Davidov & P. Schmidt, and J. Billiet (Eds.), Cross-cultural Analysis: Methods and Applications.
- Program for International Student Assessment (PISA 2003)
- 9,769 students across 40 countries on 8 quantitative math items.
- Y_{ijk} - outcome for student i, in country j and item k

$$P(Y_{ijk} = 1) = \Phi(a_{jk}\theta_{ij} + b_{jk})$$

$$a_{jk} \sim N(a_k, \sigma_{a,k}), b_{jk} \sim N(b_k, \sigma_{b,k})$$

- Both discrimination and difficulty vary across country

$$\theta_{ij} = \theta_{0j} + \varepsilon_{ij}$$

$$\theta_{0j} \sim N(0, \tau), \varepsilon_{ij} \sim N(0, v_j), \sqrt{v_j} \sim N(1, \sigma)$$

- The mean and variance of the ability vary across country
- For identification purposes the mean of $\sqrt{v_j}$ is fixed to 1
- Model preserves common measurement scale while

Cross-classified structural equation modeling

Cross-classified data

- Y_{pijk} is the $p-$th observation for person i belonging to level 2 cluster j and level 3 cluster k.
- Level 2 clusters are not nested within level 3 clusters
- Examples:
  - Natural Nesting: Students performance scores are nested within students and teachers. Students are nested within schools and neighborhoods.
  - Design Nesting: Studies where observations are nested within persons and treatments/situations.
  - Complex Sampling: Observations are nested within sampling units and another variable unrelated to the sampling.
  - Generalizability theory: Items are considered a random sample from a population of items.

Cross-classified modeling

- Why do we need to model both sets of clustering?
- Discover the true predictor/explanatory effect stemming from the clusters.
- Ignoring clustering leads to incorrect standard errors.
- Modeling with fixed effects leads to too many parameters and less accurate model.

Cross-classified model

- General SEM model: 2-way ANOVA. Y_{pijk} is the $p-$th variable for individual i in cluster j and cross cluster k

$$Y_{pijk} = Y_{1pijk} + Y_{2pj} + Y_{3pk}$$

- 3 sets of structural equations - one on each level

$$Y_{1ijk} = \nu + \Lambda_1 \eta_{ijk} + \varepsilon_{ijk}$$

$$\eta_{ijk} = \alpha + B_1 \eta_{ijk} + \Gamma_1 x_{ijk} + \xi_{ijk}$$

$$Y_{2j} = \Lambda_2 \eta_j + \varepsilon_j$$

$$\eta_j = B_2 \eta_j + \Gamma_2 x_j + \xi_j$$

$$Y_{3k} = \Lambda_3 \eta_k + \varepsilon_k$$

$$\eta_k = B_3 \eta_k + \Gamma_3 x_k + \xi_k$$

Cross - classified model

- The regression coefficients on level 1 can be a random effects from each of the two clustering levels: combines cross-classified SEM and cross classified HLM
- Bayesian MCMC estimation: used as a frequentist estimator.
- Easily extends to categorical variables.
- ML estimation possible only when one of the two level of clustering has small number of units.

Cross-classified model, example 1: Factor model

- 1 factor at the individual level and 1 factor at each of the clustering levels, 5 indicator variables on the individual level

$$y_{pijk} = \mu_p + \lambda_{1,p} f_{1,ijk} + \lambda_{2,p} f_{2,j} + \lambda_{3,p} f_{3,k} + \varepsilon_{2,pj} + \varepsilon_{3,pk} + \varepsilon_{1,pijk}.$$

- M level 2 clusters. M level 3 clusters. 1 unit within each cluster intersection. More than 1 unit is possible. Zero units possible: sparse tables.
- Estimation takes less than 1 min per replication

Cross-classified model example 1: Factor model results

Table: Absolute bias and coverage for cross-classified factor analysis model

Param	M=10	M=20	M=30	M=50	M=100
$\lambda_{1,1}$	0.07(0.92)	0.03(0.89)	0.01(0.95)	0.00(0.97)	0.00(0.91)
$\theta_{1,1}$	0.05(0.96)	0.00(0.97)	0.00(0.95)	0.00(0.99)	0.00(0.94)
$\lambda_{2,p}$	0.21(0.97)	0.11(0.94)	0.10(0.93)	0.06(0.94)	0.00(0.92)
$\theta_{2,p}$	0.24(0.99)	0.10(0.95)	0.04(0.92)	0.05(0.94)	0.02(0.96)
$\lambda_{3,p}$	0.45(0.99)	0.10(0.97)	0.03(0.99)	0.01(0.92)	0.03(0.97)
$\theta_{3,p}$	0.75(1.00)	0.25(0.98)	0.15(0.97)	0.12(0.98)	0.05(0.92)
μ_p	0.01(0.99)	0.04(0.98)	0.01(0.97)	0.05(0.99)	0.00(0.97)

Perfect coverage. Level 1 parameters estimated very well. Biases when the number of clusters is small $M = 10$. Weakly informative priors can reduce the bias for small number of clusters.

Cross-classified model, example 2: Gonzalez's example

- Gonzalez, De Boeck, Tuerlinckx (2008) A Double-Structure Structural Equation Model for Three-Mode Data. Psychological Methods, 337 - 353.
- 679 persons measured with respect to four emotional responses (frustration, tendency to act antagonistically, irritation, and anger) in a set of 11 situations
- Observations are nested within individual and situations. One observation in each crossed cell.
- The dependent variables are 4 binary outcomes decomposed as "person effect"+"situation effect"+"error"

$$y^*_{pjk} = y_{pj} + y_{pk} + \varepsilon_{pjk}$$

- Variances of ε_{pjk} is fixed to 1.

Cross-classified model, example 2: Gonzalez's example

- 2 sets of 4 random effects: 4 person effects and 4 situation effects.
- Identical structural model is estimated for the two-sets of random effects.

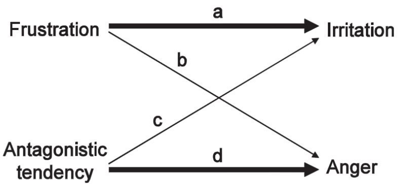

Figure 3. Graphical representation of the research questions. *a*, *b*, *c*, and *d* are effect parameters.

Cross-classified model, example 2: Gonzalez's example results

Para	M=10	M=20	M=30	M=50	M=100
β_1	0.13(0.92)	0.05(0.89)	0.00(0.97)	0.01(0.92)	0.01(0.94)
$\psi_{2,11}$	0.11(1.00)	0.06(0.96)	0.01(0.98)	0.00(0.89)	0.02(0.95)
$\psi_{2,12}$	0.15(0.97)	0.06(0.92)	0.05(0.97)	0.03(0.87)	0.01(0.96)
τ_1	0.12(0.93)	0.01(0.93)	0.00(0.90)	0.03(0.86)	0.00(0.91)

Small biases for $M = 10$. Due to parameter equalities information is combined from both clustering levels. Adding unconstrained level 1 model: tetrachoric correlation matrix.

Random loadings and variance models for cross-classified models

Cross-classified model: random loadings and interactions

- 2-way ANOVA split, Y_{pijk} is the $p-$th variable for individual i in cluster j and cross cluster k

$$Y_{pijk} = Y_{1pijk} + Y_{2pj} + Y_{3pk}$$

- 3 sets of structural equations - one on each level - with random loadings and latent variable interactions

$$Y_{1ijk} = \nu + \Lambda_1 \eta_{ijk} + \varepsilon_{ijk} + [[\eta_j * \eta_k]]$$

$$Y_{2j} = \Lambda_2 \eta_j + \varepsilon_j$$

$$Y_{3k} = \Lambda_3 \eta_k + \varepsilon_k$$

- Λ_1 can be fixed parameters or the latent variables defined on either level η_j and η_k.
- [[ ]] denote the interaction terms between Level 2 and Level 3

Cross-classified model: Alternative interaction model

- 2-way ANOVA split with interaction

$$Y_{pijk} = Y_{0pijk} + Y_{1pjk} + Y_{2pj} + Y_{3pk}$$

- Raudenbush and Bryk (2002) equation (12.3)
- The interaction term is represented by a latent variable Y_{1pjk}: the within cell (j,k) mean.
- The model needs multiple observations in cell (j,k) to be able to identify Y_{1pjk} from Y_{0pijk}
- The model is equivalent to a factor analysis model with 1 factor at level 1: Y_{1pjk}, 1 factor at level 2: Y_{2pj}, 1 factor at level 3: Y_{3pk}.

Cross-classified interaction model: Random items, Generalizability theory

- Items are random samples from a population of items.
- The same or different items may be administered to individuals.
- Suited for computer generated items and adaptive testing.
- 2-parameter IRT model

$$P(Y_{ij} = 1) = \Phi(a_j \theta_i + b_j)$$

- $a_j \sim N(a, \sigma_a)$, $b_j \sim N(b, \sigma_b)$: random discrimination and difficulty parameters
- The ability parameter is $\theta_i \sim N(0, 1)$
- Cross-classified model. Nested within items and individuals. 1 or 0 observation in each cross-classified cell.
- Interaction of two latent variables: a_j and θ_i
- The model has only 4 parameters - much more parsimonious than regular IRT models.

Random 2-parameter IRT example

- J.P. Fox (2010) Bayesian Item Response Theory. Section 4.3.3. Dutch Six Graders Math Achievement. Trends in International Mathematics and Science Study: TIMMS 2007
- 8 test items, 478 students

Table: Random 2-parameter IRT

parameter	estimate	SE
average discrimination a	0.752	0.094
average difficulty b	0.118	0.376
variation of discrimination a	0.050	0.046
variation of difficulty b	1.030	0.760

Random 2-parameter IRT example continued

- Using factor scores estimation we can estimate item specific parameter and SE using posterior mean and posterior standard deviation.

Table: Random 2-parameter IRT item specific parameters

item	discrimination	SE	difficulty	SE
Item 1	0.797	0.11	-1.018	0.103
Item 2	0.613	0.106	-0.468	0.074
Item 3	0.905	0.148	-1.012	0.097
Item 4	0.798	0.118	-1.312	0.106
Item 5	0.538	0.099	0.644	0.064
Item 6	0.808	0.135	0.023	0.077
Item 7	0.915	0.157	0.929	0.09
Item 8	0.689	0.105	1.381	0.108

Random 2-parameter IRT example comparison with ML

Table: Random 2-parameter IRT item specific parameters

item	Bayes random discrimination	Bayes random SE	ML fixed discrimination	ML fixed SE
Item 1	0.797	0.110	0.850	0.155
Item 2	0.613	0.106	0.579	0.102
Item 3	0.905	0.148	0.959	0.170
Item 4	0.798	0.118	0.858	0.172
Item 5	0.538	0.099	0.487	0.096
Item 6	0.808	0.135	0.749	0.119
Item 7	0.915	0.157	0.929	0.159
Item 8	0.689	0.105	0.662	0.134

- Bayes random estimates are shrunk towards the mean and have smaller standard errors: shrinkage estimate

Random 2-parameter IRT example continued

- One can add predictor for person's ability. For example adding gender as a predictor yields an estimate of 0.283(0.120). Males have a significantly higher math mean.
- Predictors for discrimination and difficulty random effects, for example, geometry indicator.
- More parsimonious model can yield more accurate ability estimates.

Random Rasch IRT example

- De Boeck P. (2008) Random item IRT models
- 24 verbal aggression items, 316 persons

$$P(Y_{ij} = 1) = \Phi(\theta_i + b_j)$$

$$b_j \sim N(b, \sigma)$$

$$\theta_i \sim N(0, \tau)$$

Table: Random Rasch IRT - variance decomposition

parameter	person τ	item σ	error
estimates(SE)	1.89(0.19)	1.46(0.53)	2.892
variance explained	30%	23%	46%

Random Rasch IRT example: simple model specification

```
variable: categorical = y;

analysis: type = crossedrandom;

model:

%within%

%between person%
y;

%between item%
y;
```

Application: Intensive longitudinal data

- Time intensive data: more longitudinal data are collected that makes very frequent observations using new tools for data collection. Walls & Schafer (2006)
- Typically multivariate models are developed but if the number of time points is large these models will fail due to too many variables and parameters involved
- Factor analysis models will be unstable over time. Is it lack of measurement invariance or insufficient model?
- Random loading and intercept models can take care of measurement and intercept invariance. A problem becomes an advantage.
- Random loading and intercept models produce more accurate estimates for the loadings and factors by borrowing information over time
- Random loading and intercept models produce more parsimonious model

TOCA example: Intensive longitudinal data

- Teacher-rated measurement instrument capturing aggressive-disruptive behavior among a sample of U.S. students in Baltimore public schools (Ialongo et al., 1999).
- The instrument consists of 9 items scored as 0 (almost never) through 6 (almost always).
- A total of 1174 students are observed in 41 classrooms from Fall of Grade 1 through Grade 6 for a total of 8 time points
- The multilevel (classroom) nature of the data is ignored in the current analyses.
- The item distribution is very skewed with a high percentage in the Almost Never category. The items are therefore dichotomized into Almost Never versus the other categories combined.
- We analyze the data on the original scale as continuous variables and also the dichotomized scale as categorical

TOCA example continued

- For each student a 1-factor analysis model is estimated with the 9 items at each time point
- Let Y_{pit} be the $p-$th item for individual i at time t
- We use cross-classified SEM. Observations are nested within individual and time.
- Although this example uses only 8 time points the models can be used with any number of time points.

TOCA example continued: Model 1

- Model 1: Two-level factor model with intercept non-invariance across time

$$Y_{pit} = \mu_p + \zeta_{pt} + \xi_{pi} + \lambda_p \eta_{it} + \varepsilon_{pit}$$

- μ_p, λ_p are model parameters, $\varepsilon_{pit} \sim N(0, \theta_{w,p})$ is the residual
- $\zeta_{pt} \sim N(0, \sigma_p)$ is a random effect to accommodate intercept non-invariance across time
- To correlate the factors η_{it} within individual i

$$\eta_{it} = \eta_{b,i} + \eta_{w,it}$$

- $\eta_{b,i} \sim N(0, \psi)$ and $\eta_{w,it} \sim N(0, 1)$. The variance is fixed to 1 to identify the scale in the model.
- $\xi_{pi} \sim N(0, \theta_{b,p})$ is a between level residual in the between level factor model
- Without the random effect ζ_{pt} this is just a standard two-level factor model.

TOCA example continued: Model 2

- Model 2: Adding latent growth model for the factor

$$\eta_{it} = \alpha_i + \beta_i * t + \eta_{w,it}$$

- $\alpha_i \sim N(0, v_\alpha)$ is the intercept and $\beta_i \sim N(0, v_\beta)$ is the slope. For identification purposes again $\eta_{w,it} \sim N(0,1)$.
- The model looks for developmental trajectory across time for the aggressive-disruptive behavior factor
- Such a trend trajectory much less likely to hold across the entire population, i.e., the model parameters μ_p can be restricted through a linear trend but much less likely to hold true(μ_p have small SE and linear trend will be rejected).

TOCA example continued: Model 3

- Model 3: Adding measurement non-invariance.
- Replace the fixed loadings λ_p with random loadings $\lambda_{pt} \sim N(\lambda_p, w_p)$
- The random loadings accommodate measurement non-invariance across time.
- We estimate Model 3 for continuous and categorical scale on the TOCA data

TOCA example continued: Results for continuous analysis.

Number of Free Parameters 56

MODEL RESULTS

	Estimate	Posterior S.D.	One-Tailed P-Value	95% C.I. Lower 2.5%	Upper 2.5%	Significance
Within Level						
Residual Variances						
Y1	1.073	0.022	0.000	1.029	1.119	*
Y9	0.630	0.014	0.000	0.604	0.658	*
E	1.000	0.000	0.000	1.000	1.000	*
Between ID Level						
Variances						
Y1	0.146	0.016	0.000	0.118	0.180	*
Y9	0.052	0.009	0.000	0.035	0.068	*
I	1.316	0.080	0.000	1.172	1.486	*
S	0.026	0.003	0.000	0.020	0.032	*
Between T1 Level						
Means						
Y1	1.632	0.120	0.000	1.377	1.885	*
Y9	1.232	0.096	0.000	1.044	1.420	*
S1	0.679	0.023	0.000	0.640	0.732	*
S9	0.705	0.043	0.000	0.628	0.797	*
Variances						
Y1	0.080	0.138	0.000	0.025	0.372	*
Y9	0.047	0.109	0.000	0.017	0.266	*
S1	0.002	0.004	0.000	0.000	0.013	*
S9	0.010	0.079	0.000	0.003	0.052	*

TOCA example continued: Results for categorical analysis.

Number of Free Parameters 47

MODEL RESULTS

	Estimate	Posterior S.D.	One-Tailed P-Value	95% C.I. Lower 2.5%	95% C.I. Upper 2.5%	Significance
Within Level						
Residual Variances						
E	1.000	0.000	0.000	1.000	1.000	*
Between ID Level						
Variances						
Y1	0.153	0.031	0.000	0.098	0.221	*
Y9	0.201	0.049	0.000	0.114	0.309	*
I	1.074	0.078	0.000	0.932	1.242	*
S	0.024	0.003	0.000	0.018	0.031	*
Between T1 Level						
Means						
S1	1.003	0.066	0.000	0.885	1.142	*
S9	1.352	0.112	0.000	1.147	1.588	*
Thresholds						
Y1$1	-0.850	0.148	0.000	-1.144	-0.561	*
Y9$1	-0.483	0.124	0.001	-0.744	-0.242	*
Variances						
Y1	0.100	0.221	0.000	0.029	0.574	*
Y9	0.073	0.121	0.000	0.019	0.408	*
S1	0.012	0.044	0.000	0.001	0.100	*
S9	0.045	0.133	0.000	0.006	0.325	*

Conclusion

- Bayesian methods solve problems not feasible with ML or WLS
- Mplus Version 7 includes also 3-level SEM/MHLM for continuous and categorical with ML and Bayes
- Mplus Version 7: August 2012
- Mplus short courses Utrecht August 27-29